

Homework problems:

11.1 (due Nov 21, 2006) Professor A designed a black-box which on input a computes a^2 in time $O(\log a)$. We would like to use the black-box to multiply numbers, i. e., on input a, b we want to compute ab . We want our algorithm to run in time $O(\log(ab))$.

- a) Give such an algorithm.
- b) Suppose now, that instead of $x \mapsto x^2$ black-box, we have $x \mapsto x^3$ black-box. Show how we can use the new black-box to multiply numbers a, b in time $O(\log(ab))$.
- c) Suppose now, that instead of $x \mapsto x^2$ black-box, we have $x \mapsto x^4$ black-box. Show how we can use the new black-box to multiply numbers a, b in time $O(\log(ab))$.

In parts a), b), c) you can assume that we can add two numbers c, d in $O(\log(cd))$ -time. You can also assume that for any constant f we can divide d by f in $O(\log d)$ -time.

11.2 (due Nov 21, 2006) Let x, y be unknown positive integers. Let $A = xy$ and $B = x + y$. Give a polynomial-time algorithm which on input A, B computes x, y .