

Homework problems:

3.1 (due Sep 28, 2006) We are given an array of integers $A[1..n]$. We would like to determine whether there exists an integer x which occurs in A more than $n/2$ times (i. e., whether A has a majority element). Give an algorithm which runs in time $O(n)$.

Example: For $A = [3, 1, 2]$ the answer is NO. For $A = [3, 1, 3]$ the answer is YES.

3.2 (due Sep 28, 2006) We are given an array of integers $A[1..n]$. We would like to determine whether there exists an integer x which occurs in A more than $n/3$ times. Give an algorithm which runs in time $O(n)$.

3.3 (due Sep 28, 2006) We are given an array of integers $A[1..n]$ which is almost sorted in the following sense: for all $i \in \{1, \dots, n - k\}$ we have $A[i] \leq A[i + k]$. Give an algorithm which sorts the array A . Your algorithm should run in time $O(n \log k)$.

3.4 (due Sep 28, 2006) We are given two arrays of integers $A[1..n]$ and $B[1..n]$, and a number X . Design an algorithm which decides whether there exist $i, j \in \{1, \dots, n\}$ such that $A[i] + B[j] = X$. Your algorithm should run in time $O(n \log n)$.

Bonus problem:

3.5 (due Sep 28, 2006) There is a very long 1-dimensional table in CSB 714 $\frac{1}{2}$. Unfortunately the table has lots of holes. The wizard living in the room wants to cover the holes using two tableclothes, minimizing the total amount of material used. The positions of the holes are given by a set of n numbers $A = \{a_1, \dots, a_n\}$. The goal is to find two intervals $[c, d]$ and $[e, f]$ such that $A \subseteq [c, d] \cup [e, f]$ (i. e., the intervals cover the holes), and such that the total length of the intervals (i. e., $d - c + f - e$) is minimized. Give an $O(n)$ -time algorithm which finds the intervals.

Example: For $A = \{90, 10, 20, 40, 70, 60, 50, 80\}$ the answer is $[10, 20]$, $[40, 90]$.