

Linear programming (part 3 of CSC 282),

<http://www.cs.rochester.edu/~stefanko/Teaching/12CS282>

1 Schedule

"Homework" problem session is in CSB 601, 5-6pm on Nov. 7 (Wednesday); held by Julian Lunger.

Homework is **due Nov 8** (Thursday).

"Exam" problem session is in CSB 601, 5-6pm on Nov. 12 (Monday).

EXAM #3 will be on **Tuesday, Nov. 13**.

Concepts and methods that will be tested on the exam include the following:

- formulating a linear program for a problem (similar to problems 3.4, 3.6, and 3.7 below),
- duality (similar to problem 3.3 below),
- zero-sum games (similar to problems 3.1 and 3.2 below).

2 Homework - solve and turn in

3.1 (due Nov 8) Compute the value of the zero-sum two person game given by the following payoff matrix:

$$\begin{pmatrix} 2 & 4 \\ 1 & 3 \\ 4 & 2 \end{pmatrix}$$

3.2 (due Nov 8) Consider the zero-sum game with the following pay-off matrix

$$\begin{pmatrix} x & -3 \\ -2 & 4 \end{pmatrix}.$$

For what value of x is the value of the game 0? Clearly explain how you obtained the answer.

3.3 (due Nov 8) Construct the linear program dual to the following linear program:

$$\begin{aligned} \max x_1 + 2x_2 + 3x_3 + 4x_4 \\ x_2 + x_3 + x_4 &\leq 1 \\ x_1 + x_3 + x_4 &\leq 2 \\ x_1 + x_2 + x_4 &\leq 3 \\ x_1 + x_2 + x_3 &\leq 4 \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0. \end{aligned}$$

Find the optimal solution of the primal and the dual problem. **Use a linear programming solver** to obtain the solutions (for example you can use freeware `lpsolve` or function `Maximize` in MATHEMATICA (installed in some labs)).

3.4 (due Nov 8) There are n bottles which contain different mixtures of three chemicals called A, B, C . The i -th bottle contains the chemicals in ratio $a_i : b_i : c_i$ (thus, $a_i/(a_i + b_i + c_i)$ fraction of the i -th bottle is chemical A , $b_i/(a_i + b_i + c_i)$ fraction of the i -th bottle is chemical B , and $c_i/(a_i + b_i + c_i)$ fraction of the i -th bottle is chemical C). We want to know whether it is possible to obtain a mixture containing the chemicals A, B, C in ratio $a : b : c$ by mixing various amounts from the bottles. Give an efficient algorithm for this problem.

For example, if the input is $n = 2$, the ratios in the bottles are $1 : 1 : 2$ and $3 : 3 : 1$, and we want to obtain mixture with ratio $1 : 1 : 1$ then the answer is YES (we can take 2 parts from the first bottle and 1 part from the second bottle).

3.5 (due Nov 8) Let a be a constant and x be a variable. What is the value of the following linear program? Give a simple mathematical expression.

$$\begin{aligned} \min x \\ x &\geq a \\ x &\geq -a \end{aligned}$$

Let a, b be constants and x, y be variables. What is the value of the following linear program? Give a simple mathematical expression.

$$\begin{aligned} \min x + y \\ x &\geq a \\ x &\geq -a \\ y &\geq b \\ y &\geq -b \end{aligned}$$

(If you don't know where to start—plug in concrete values for a and b (make sure to use negative numbers too), solve the resulting linear program (either by hand or using a solver) and formulate a conjecture based on the experiments.)

Let a, b be constants and x, y be variables. What is the value of the following linear program? Give a simple mathematical expression.

$$\begin{aligned} \min x - y \\ x &\geq a \\ x &\geq -a \\ y &\geq b \\ y &\geq -b \end{aligned}$$

3 Bonus Homework - solve and turn in

3.6 (due Nov 8) We are given n pairs of real numbers $(x_1, y_1), \dots, (x_n, y_n)$. We want to find real numbers a, b which minimize

$$\max_{i \in \{1, \dots, n\}} |ax_i + y_i - b|.$$

Write a linear program for this problem.

3.7 (due Nov 8) We are given n pairs of real numbers $(x_1, y_1), \dots, (x_n, y_n)$. We want to find real numbers a, b which minimize

$$\sum_{i \in \{1, \dots, n\}} |ax_i + y_i - b|.$$

Write a linear program for this problem.

3.8 (due Nov 8) We are given n pairs of real numbers $(x_1, y_1), \dots, (x_n, y_n)$. We want to find real numbers a, b which minimize

$$\sum_{i \in \{1, \dots, n\}} (ax_i + y_i - b)^2.$$

Find a formula for a and b (you do NOT need linear programming for this one).

4 Additional problems from the book (do not turn in)

Try to solve the following problems. A few of them may be on the quiz. We will go over the ones that you choose in the problem sessions.

- 7.1, 7.2, 7.3, 7.4, 7.6, 7.7, 7.8, 7.9, 7.11, 7.12, 7.13, 7.14, 7.15, 7.19, 7.27, 7.28.