

## 1 Schedule

"Homework" problem session is in CSB 601, 5-6pm on Nov. 28 (Wednesday); held by Julian Lunger.

Homework is **due Nov 29** (Thursday).

"Exam" problem session is in CSB 601, 5-6pm on Dec. 3 (Monday).

**EXAM #3** will be on **Tuesday, Dec. 4**.

## 2 Homework - solve and turn in

**4.1 (due Nov 29)** We are given a string  $S$  and a number  $k$ . We want to find a string that appears in  $S$  exactly  $k$  times (if such a string exists). Give a linear time algorithm for the problem.

**4.2 (due Nov 29)** We are given a string  $S$ . We want to find the longest string  $Z$  such that  $S$  contains (at least) two non-overlapping copies of  $Z$ . Give a linear time algorithm for the problem.

**4.3 (due Nov 29)** Let  $A$  and  $B$  be two sets. Their sum  $A + B$  is defined to be

$$A + B := \{a + b \mid a \in A, b \in B\},$$

for example if  $A = \{1, 4, 6\}$  and  $B = \{2, 5\}$  then  $A + B = \{3, 6, 8, 9, 11\}$ . Give  $O(n \log n)$  algorithm whose input is two sets  $A, B \subseteq \{1, \dots, n\}$  and the output is set  $A + B$ .

**4.4 (due Nov 29)** The Hadamard matrices  $H_0, H_1, \dots$ , are defined as follows.

- $H_0$  is the  $1 \times 1$  matrix  $[1]$ .
- For  $k > 0$ ,  $H_k$  is the  $2^k \times 2^k$  matrix

$$H_k = \left[ \begin{array}{c|c} H_{k-1} & H_{k-1} \\ \hline H_{k-1} & -H_{k-1} \end{array} \right].$$

Show that if  $v$  is a column vector of length  $n = 2^k$  then the matrix-vector product  $H_k v$  can be calculated using  $O(n \log n)$  operations. (Assume that all the numbers involved are small enough that basic arithmetic operations take  $O(1)$  time.)