
CSC 284/484 Advanced Algorithms - applied homework 1A
due: January 25th, before class

Definition 0.1 Let $G = (V, E)$ be a directed graph. Let $s, t \in V$ be two distinct vertices. An s - t cut is a pair of disjoint sets A, B such that $s \in A$, $t \in B$, and $A \cup B = V$.

We will prove

Theorem 0.2 Let $G = (V, E)$ be a directed graph with edge capacities $c : E \rightarrow \mathbb{R}_{\geq 0}$. Let $s, t \in V$ be two distinct vertices. We have

$$\max_{s-t \text{ flows } f} |f| = \min_{s-t \text{ cuts } A, B} \text{capacity}(A, B).$$

Definition 0.3 Let $G = (V, E)$ be a directed graph. Let $s, t \in V$ be two distinct vertices. An s - t alternative-cut is a pair of disjoint sets A, B such that $s \in A$, $t \in B$, $A \cup B = V$, every vertex in A is reachable from s using paths contained in A , and every vertex in B can reach t using paths contained in B .

Prove or disprove

Conjecture 1 Let $G = (V, E)$ be a directed graph with edge capacities $c : E \rightarrow \mathbb{R}_{\geq 0}$. Let $s, t \in V$ be two distinct vertices. Assume that every $v \in V$ is reachable from s . Assume that t can be reached from every $v \in V$. We have

$$\max_{s-t \text{ flows } f} |f| = \min_{s-t \text{ alternative-cuts } A, B} \text{capacity}(A, B).$$